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NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

For Supervisor's use only

Level 3 Calculus, 2008

90635 Differentiate functions and use derivatives to solve problems

Credits: Six
9.30 am Tuesday 18 November 2008

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables Booklet L3–CALCF.

You should answer ALL the questions in this booklet.

Show ALL working for ALL questions.

Show any derivatives that you need to find when solving the problems.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only	**Achievement Criteria**	
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Differentiate functions and use derivatives to solve problems. ☐	Demonstrate knowledge of advanced concepts and techniques of differentiation and solve differentiation problems. ☐	Solve more complex differentiation problem(s). ☐
Overall Level of Performance ☐		

You are advised to spend 50 minutes answering the questions in this booklet.

Assessor's use only

QUESTION ONE

Differentiate the following functions.
You do not need to simplify your answers.

(a) $y = \sqrt{x^2 - x}$

(b) $y = \dfrac{\sin 3x}{x^3 - x}$

QUESTION TWO

Assessor's use only

The height of a seedling at the end of its first month is given by:

$$H(w) = 12w - 1.2e^{0.2w} \text{ for } 0 < w < 25$$

where H is the height, in cm, of the seedling at the end of the month,
and w is the amount of water the seedling receives (litres per m^2 of soil surface).

Find the amount of water required for the seedling to grow to the maximum height in its first month.

You may assume that $\dfrac{d^2H}{dw^2} < 0$

Give any derivatives that you need to find when solving this problem.

QUESTION THREE

Assessor's use only

An island has a population of possums which are being trapped.
The number of possums on the island is given by

$$P(t) = 5000 - 4t^2 - 1500\ln(2t + 1), \qquad 0 < t < 10$$

where P is the number of the possums,
and t is the time in months after the trapping begins.

What is the rate of change in the population of possums after 3 months of trapping?
Give any derivatives that you need to find when solving this problem.

QUESTION FOUR

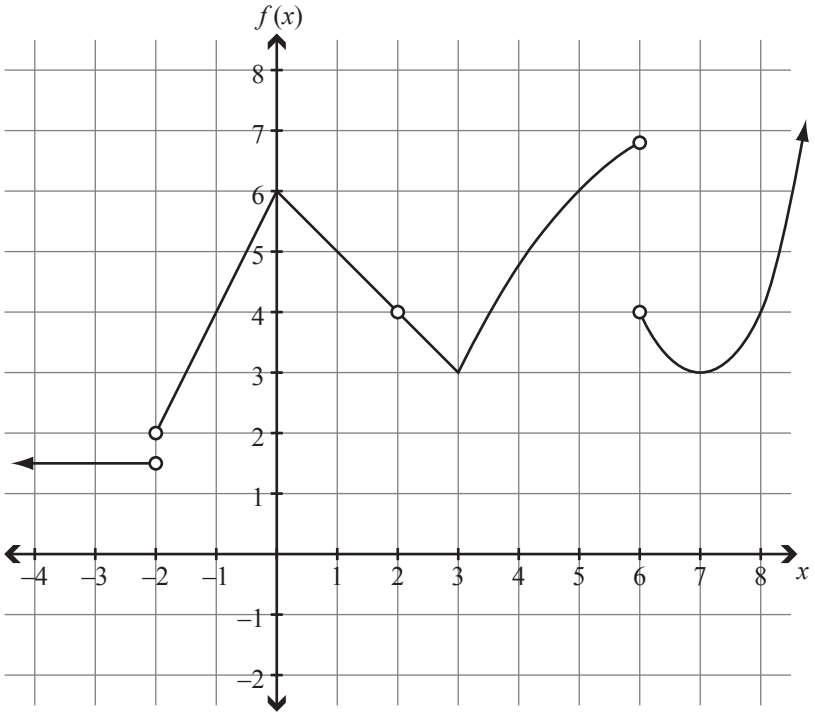
Assessor's use only

A curve is defined parametrically by $x = 2\tan t$ and $y = 3\sin 2t$.

Find the gradient of the curve at $t = \frac{\pi}{6}$.

QUESTION FIVE

Assessor's use only



The graph above defines the function $y = f(x)$.

(a) For the function $f(x)$ above, find all the values of a where $\lim_{x \to a} f(x)$ does not exist.

(b) For the function $f(x)$ above, list all the x values that meet the following conditions.

(i) $f'(x) = 0$

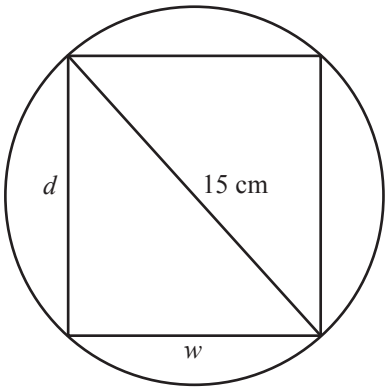
(ii) $f''(x) > 0$

(iii) $f(x)$ is continuous but not differentiable.

QUESTION SIX

Assessor's use only

A rectangular beam of width w cm and depth d cm is cut from a cylindrical pine log.
The diameter of the cross section of the log is 15 cm.



The strength S of the beam is proportional to the product of its width and the square of its depth.

$S = kd^2w$, where k is a constant.

What dimensions will give the beam maximum strength?

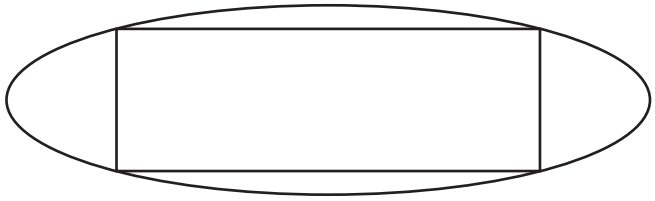
You may assume that the second derivative is negative.
Give any derivatives that you need to find when solving this problem.

QUESTION SEVEN

Assessor's use only

An ellipse has the equation $\frac{x^2}{9} + y^2 = 1.$

A rectangle is inside the ellipse with its vertices on the ellipse as shown.



Find the maximum area of the rectangle.

You may assume $\frac{\mathrm{d}^2 A}{\mathrm{d}x^2} < 0$ and $\frac{\mathrm{d}^2 A}{\mathrm{d}y^2} < 0$ where A is the area of the rectangle.

Give any derivatives that you need to find when solving this problem.

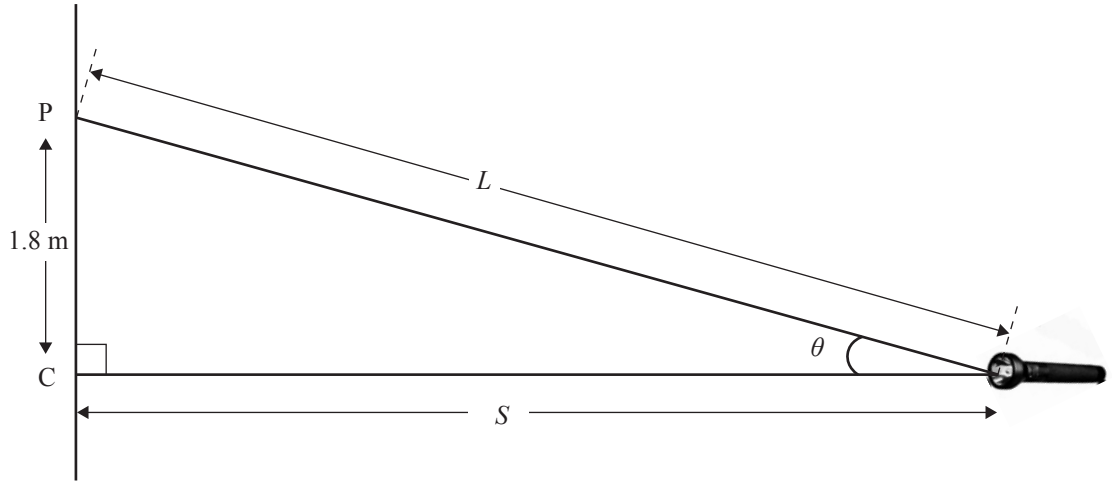
Assessor's use only

Question Eight is on the following page.

QUESTION EIGHT

Assessor's use only

A man is walking in the dark with a torch.
He is walking directly towards point C on a wall, with the torch held horizontally aimed at point C.
Point P is 1.8 m directly above point C.
The man is walking at 0.6 m s^{-1}.



The intensity of the light at point P is given by the formula

$$I = \frac{600\cos\theta}{L^2}$$

where L is the distance from the torch to point P in metres,
and I is the intensity of light, measured in lux, at point P.

What is the rate at which the intensity of light is changing when the torch is 2.4 m from point P?
Give any derivatives that you need to find when solving this problem.

Assessor's use only

Extra paper for continuation of answers if required.
Clearly number the question.

Assessor's use only

Question number

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